3. Kinematics

3.1 Forward Kinematics

- Let's use the Robotics Toolbox to review the last example: “RRP_3DOF_example1.m”

% Define the D-H table for the RRP robot
% alpha0 = 0; a0=0; d1=3; theta1=0;
L(1) = Link([theta1 d1 a0 alpha0 0 0], 'modified');
% We use the modified DH

alpha1 = -90*pi/180; a1=1; d2=2; theta2=0;
L(2) = Link([theta2 d2 a1 alpha1 0 -90*pi/180], 'modified'); % We use the modified DH

alpha2 = 90*pi/180; a2=0; d3=0; theta3=0;
L(3) = Link([theta3 d3 a2 alpha2 1 2], 'modified'); % We use the modified DH

RRP_robot = SerialLink(L, 'name', 'RRP Robot')
...
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- Add link frames: “RRP_3DOF_example2.m”

Frame {4} “Tool Frame”
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Example:

- An alternative solution to the last example.
  
  - Noting that some of the choices made when affixing the link coordinate frames were not unique, it seems reasonable to see if there might be alternatives which might lead to a simpler D-H table.
  
  - Let us first redraw the manipulator in the “kinematic home position”. This basically refers to the configuration of the robot wherein all the joint variables \( \theta_i \)'s (and \( d_i \)'s) are set to zero (i.e., \( \dot{\theta}(t) = 0 \)) and all offsets are “unwound”.
  
  - Also, let us choose to place frame \{0\} to be coincident with frame \{1\} when the first joint variable \( \theta_1 = 0 \).
  
  - Frame \{4\} is unnecessary as frame \{3\} is also fixed \( \text{wrt} \) the gripper. Furthermore, frame \{3\} can be chosen such that the z-axis offset between \{2\} and \{3\} is zero (i.e., \( \dd{d}_3 = 0 \)).
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The Denavit-Hartenberg table:

<table>
<thead>
<tr>
<th>i</th>
<th>D-H params.</th>
<th>$\alpha_{i-1}$</th>
<th>$a_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_1(t)$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>$-90$</td>
<td>$a_1$</td>
<td>$d_2$</td>
<td>$\theta_2(t)$</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>90</td>
<td>0</td>
<td>$d_3(t)$</td>
<td>0</td>
</tr>
</tbody>
</table>

- We can again generate the “T-matrices

Next, generate the “T-matrices”
- Courtesy of Mathematica
  - Thur 13 Sept Example.pdf
Let us now use the D-H parameters from the link table to construct the homogeneous matrices which will relate one link coordinate frame to the next.

\[ ^0T(\alpha_0, a_0, d_1, \theta_1(t)) = ^0T(0,0,0, \theta_1(t)) = \begin{bmatrix}
    C_1 & -S_1 & 0 & 0 \\
    S_1 & C_1 & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix} \]

\[ ^1T(\alpha_1, a_1, d_2, \theta_2(t)) = ^1T(-90, a_1, d_2, \theta_2(t)) = \begin{bmatrix}
    C_2 & -S_2 & 0 & a_1 \\
    S_2 & C_2 & 0 & 0 \\
    0 & 0 & 1 & d_2 \\
    0 & 0 & 0 & 1
\end{bmatrix} \]

\[ ^2T(\alpha_2, a_2, d_3(t), \theta_3) = ^2T(90,0, d_3(t), 0) = \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 0 & -1 & -d_3 \\
    0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix} \]
Therefore, we can now compute the coordinates of link 3 (frame \{3\}) as seen from the base or inertial (frame \{0\}) frame of reference, in terms of the 3 joint variables \((\theta_1, \theta_2, d_3)\).

\[
^{0}T_{3} = ^{0}T_{1}^{1}T_{2}^{2}T_{3}^{3}
\]

\[
= \begin{bmatrix}
C_1C_2 & -S_1 & C_1S_2 & -d_2S_1 + C_1(a_1 + d_3S_2) \\
C_2S_1 & C_1 & S_1S_2 & C_1d_2 + S_1(a_1 + d_3S_2) \\
-S_2 & 0 & C_2 & C_2d_3 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(3.3)

Compare the expression we now have via equation (3.3) with the expression we previously obtained in equation (3.2). This expression is obviously simpler!!
Example:

- A more advanced example, the NG Remotec Andros Wolverine EOD (explosive ordinance disposal) mobile manipulator.

- In particular, we will be looking at the arm in its “kinematic home position” (i.e., all the joint variables $\theta_i$'s (and $d_i$'s) are set to zero).
i. Identify the **joint axes** and draw ∞ lines along them.
ii. Assign the link \((i - 1)^{th}\) coordinate frame origin at the point along the axis \(i - 1\) where the mutual perpendicular (between axis \(i - 1\) and axis \(i\)) meets it.

**Note:** Since there is no mutual perpendicular between axis 6 and axis 7 (non-existent), the origin of frame \{6\} can be placed anywhere on axis 6.
iii. Assign the $\hat{Z}_{i-1}$ direction along the $i - 1^{th}$ axis.

**Note:** The direction of z-axis is not unique.
iv. Assign the $\hat{x}_{i-1}$ axis along the mutual perpendicular, from $\hat{z}_{i-1}$ to $\hat{z}_i$.

For $i = 2$, $\hat{x}_1$ axis from $\hat{z}_1$ to $\hat{z}_2$ (not unique since axis 1 and axis 2 intersect).
iv. Assign the $\hat{x}_{i-1}$ axis along the mutual perpendicular, from $\hat{z}_{i-1}$ to $\hat{z}_i$.

For $i = 3$, $\hat{x}_2$ axis from $\hat{z}_2$ to $\hat{z}_3$ (unique).
iv. Assign the $\hat{x}_{i-1}$ axis along the mutual perpendicular, from $\hat{z}_{i-1}$ to $\hat{z}_i$.

For $i = 4$, $\hat{x}_3$ axis from $\hat{z}_3$ to $\hat{z}_4$ (unique).
iv. Assign the $\hat{x}_{i-1}$ axis along the **mutual perpendicular**, from $\hat{z}_{i-1}$ to $\hat{z}_i$.

For $i = 5$, $\hat{x}_4$ axis from $\hat{z}_4$ to $\hat{z}_5$ (not unique since axis 4 and axis 5 intersect).
iv. Assign the $\hat{x}_{i-1}$ axis along the mutual perpendicular, from $\hat{z}_{i-1}$ to $\hat{z}_i$.

For $i = 6$, $\hat{x}_5$ axis from $\hat{z}_5$ to $\hat{z}_6$ (not unique since axis 5 and axis 6 are colinear).
iv. Assign the $\hat{x}_{i-1}$ axis along the mutual perpendicular, from $\hat{z}_{i-1}$ to $\hat{z}_i$.

For $i = 7$, $\hat{x}_6$ axis from $\hat{z}_6$ to $\hat{z}_7$ (not unique since axis 7 does not exist).
v. Include the \( \hat{y}_{i-1} \) axis to complete the coordinate system. Recall: Right-hand coordinate system.

vi. Add a base frame \( \{0\} \) and a tool frame (if desired). Note: Must adhere to the two D-H constraints when choosing frames \( \{0\} \)!
Next, generate the "T-matrices"

- Courtesy of Mathematica
  - Andros Example.pdf
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Note:

1. The robot is not truly in the “kinematic home position” as \( d_5(t) \) is not equal to zero.

2. The joint displacements \( d_1 \) and \( d_6 \) could be set to zero, however, the coordinate frames would look cluttered.

Let us now use the D-H parameters from the link table to construct the homogeneous matrices which will relate one link coordinate frame to the next.

\[
^{0}T(\alpha_0, a_0, d_1, \theta_1(t)) = ^{0}T(0,0,d_1,\theta_1(t)) = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
^{1}T(\alpha_1, a_1, d_2, \theta_2(t)) = ^{1}T(-90,0,0,\theta_2(t)) = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -S_2 & -C_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]
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\[ 2^3 T(\alpha_2, a_2, d_3(t), \theta_3(t)) = 2^3 T(0, a_2, 0, \theta_3(t)) = \begin{bmatrix} C_3 & -S_3 & 0 & a_2 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ 3^4 T(\alpha_3, a_3, d_4, \theta_4(t)) = 3^4 T(0, a_3, 0, \theta_4(t)) = \begin{bmatrix} C_4 & -S_4 & 0 & a_3 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ 4^5 T(\alpha_4, a_4, d_5(t), \theta_5) = 4^5 T(90, 0, d_5(t), 0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]
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Therefore, we can now compute the coordinates of the gripper (frame \{6\}) as seen from the base frame (i.e. frame \{0\}) of reference, in terms of the 6 joint variables \((\theta_1, \theta_2, \theta_3, \theta_4, d_5, \theta_6)\).

\[
{^5}_6T(\alpha_5, a_5, d_6, \theta_6(t)) = {^5}_6T(0, 0, d_6, \theta_6(t)) \quad = \quad \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

\[
{^0}_6T = {^0}_1T{^1}_2T{^2}_3T{^3}_4T{^4}_5T{^5}_6T
\]

\[
\begin{bmatrix} C_1C_6C_{234} - S_1S_6 & -C_6S_1 - C_1C_{234}S_6 & C_1S_{234} & C_1 \left( a_2C_2 + a_3C_{23} + (d_5 + d_6)S_{234} \right) \\ C_6C_{234}S_1 + C_1S_6 & C_1C_6 - C_{234}S_1S_6 & S_1S_{234} & S_1 \left( a_2C_2 + a_3C_{23} + (d_5 + d_6)S_{234} \right) \\ -C_6S_{234} & S_6S_{234} & C_{234} & d_1 + C_{234} \left( d_5 + d_6 \right) - a_2S_2 - a_3S_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]