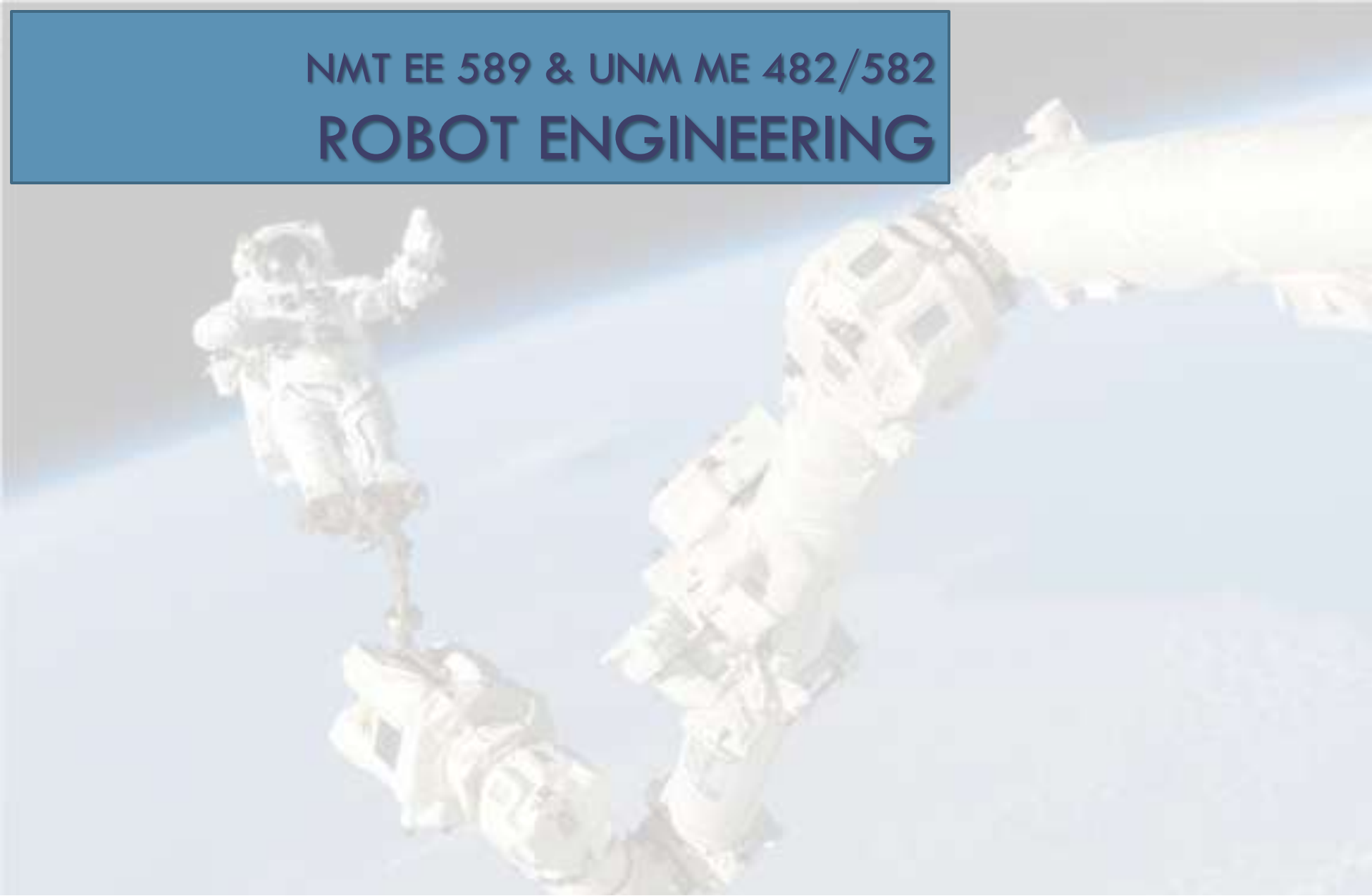
NMT EE 589 & UNM ME 482/582

ROBOT ENGINEERING



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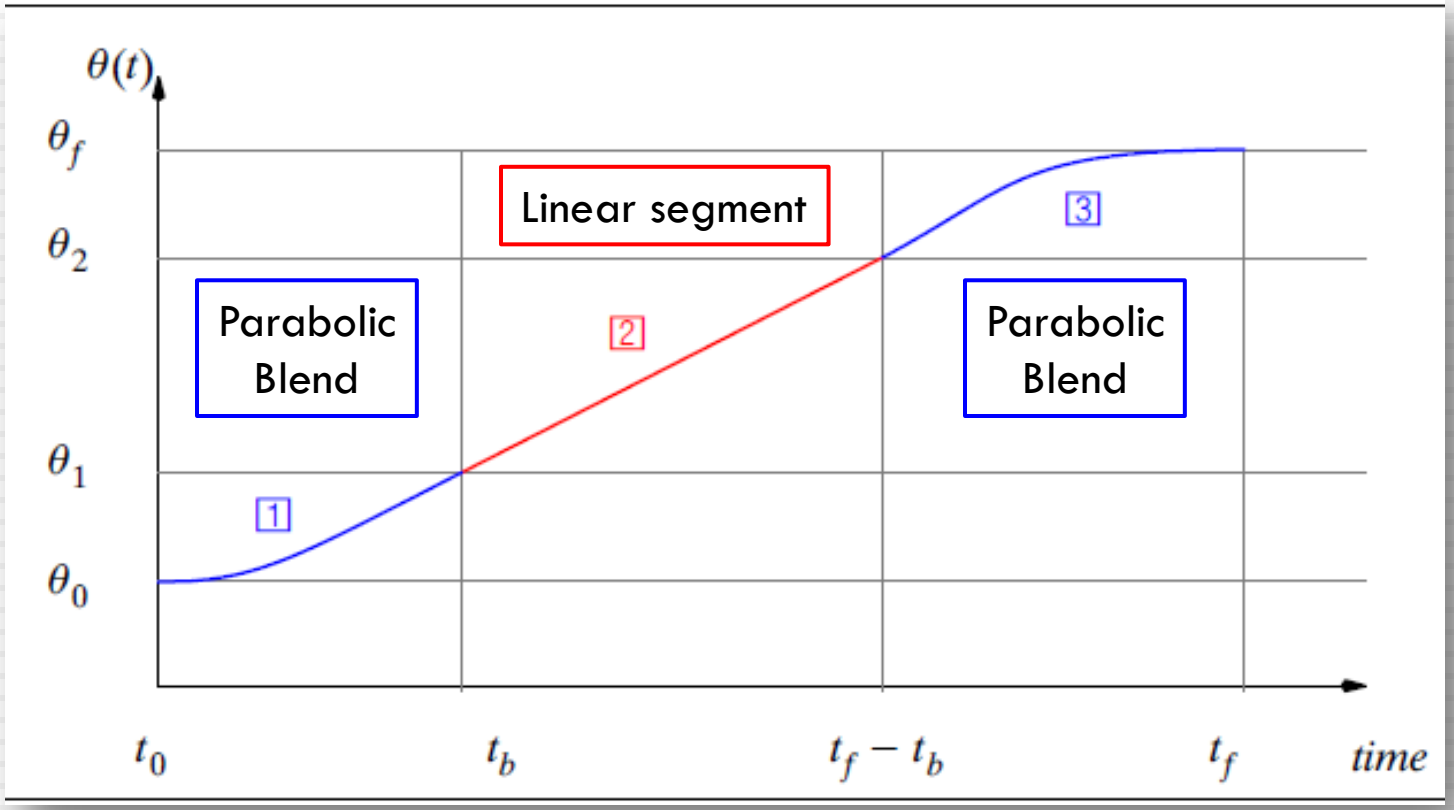
NMT EE 589 & UNM ME 482/582

6.3.3 A Linear Segment With Parabolic Blends

- **It is often desirable to minimize the time to traverse a joint trajectory.**
  - Would like to move at a max constant joint rate.
  - To minimize time employ max joint acceleration to get to this constant joint rate



- **Assume that we start from a position $\theta(t_0) \doteq \theta_0$ and come to rest at a final point $\theta(t_f) \doteq \theta_f$.**
- **Furthermore, assume that the manipulator was initially at rest $\dot{\theta}(t_0) = 0$, and that it will come to rest at the final point $\dot{\theta}(t_f) = 0$.**
  - Let's allow $t_0 = 0$, for simplicity's sake!! (can always compensate afterwards)

Segment $\boxed{1}$:

- **Assume a constant acceleration of $\ddot{\theta}(t) \doteq a$. Using a Taylor series expansion:**

$$\theta(t) = \theta_0 + \cancelto{0}{\dot{\theta}(t_0)}[t - t_0] + \ddot{\theta}(t_0)\frac{[t - t_0]^2}{2} \quad , t_0 \le t \le t_b$$

$$= \theta_0 + a\frac{t^2}{2} \quad , t \in [0, t_b)$$

- **Thus, the position at the end of this segment is**

$$\theta(t_b) = \theta_0 + a\frac{t_b^{\,2}}{2} \doteq \theta_1 \qquad \textbf{(6.5)}$$

- **The velocity at any time during this segment is**

$$\dot{\theta}(t) = at$$

- **Thus, the velocity at the end of this blend segment is**

$$\dot{\theta}(t_b) = a\,t_b \doteq v \quad \textbf{(say)} \qquad \textbf{(6.6)}$$

Segment $\boxed{2}$:

- **A constant velocity of** $\dot{\theta}(t) \doteq v$

$$\theta(t) = \theta_1 + \dot{\theta}(t_b)[t - t_b] \quad , \quad t_b \le t \le t_f - t_b$$

$$\Rightarrow \qquad \theta(t_f - t_b) = \theta_1 + [t_f - 2t_b]v \doteq \theta_2 \qquad \textbf{(6.7)}$$

- **and**

$$\dot{\theta}(t) = \dot{\theta}(t_b) = v \quad , t_b \le t \le t_f - t_b$$

Segment $\boxed{3}$:

- **A constant acceleration of $\ddot{\theta}(t) \doteq -a$**

$$\theta(t) = \theta_2 + \dot{\theta}(t_f - t_b)[t - [t_f - t_b]] + \ddot{\theta}(t_f - t_b)\frac{[t - [t_f - t_b]]^2}{2} \quad , (t_f - t_b) \le t \le t_f$$

$$= \theta_2 + v[t - [t_f - t_b]] - a\frac{[t - [t_f - t_b]]^2}{2}$$

- **Thus, the position at the end of this segment is**

$$\theta(t_f) = \theta_2 + vt_b - a\frac{t_b^{\,2}}{2} \doteq \theta_f \qquad \textbf{(6.8)}$$

- **and the velocity at the end of this blend segment is**

$$\dot{\theta}(t)\Big|_{t=t_f} = v - a[t - [t_f - t_b]]\Big|_{t=t_f}$$

$$= v - at_b = 0$$

- Note that: Given θ_0, θ_f, t_0, and t_f, if we choose to decrease the magnitude of the acceleration ($|a|$) then segments $\boxed{1}$ and $\boxed{3}$ become larger and segment $\boxed{2}$ becomes smaller.
- If we choose the magnitude of the acceleration during the blend periods to be too small, then a viable solution may cease to exist!!

- **Question:**
  - What is the bound on the value of the acceleration for a viable LSPB?
- **Let's use equations (6.5–6.8) to develop a relationship between t_b and the acceleration a. Substituting for θ_2 in equation (6.8) using equation (6.7) we get,**

$$\theta_f = \theta_1 + v[t_f - 2t_b] + vt_b - a\frac{t_b^2}{2}$$

$$\theta_f = \theta_2 + vt_b - a\frac{t_b^2}{2}$$

$$\theta_2 = \theta_1 + [t_f - 2t_b]v$$

- **then, substituting for θ_1 using (6.5) we get,**

$$\theta_f = \theta_0 + a\frac{t_b^2}{2} + v[t_f - 2t_b] + vt_b - a\frac{t_b^2}{2}$$
$$= \theta_0 + v[t_f - t_b]$$

$$\theta_1 = \theta_0 + a\frac{t_b^2}{2}$$

- **Recalling the expression for v from equation (6.6) gives**

$$\theta_f = \theta_0 + a t_b [t_f - t_b] \qquad v = a t_b$$

$$\Rightarrow \qquad a t_b^{\,2} - a t_f t_b + [\theta_f - \theta_0] = 0$$

- **Solving for this quadratic in t_b gives**

$$t_b = \frac{a t_f \pm \sqrt{a^2 t_f^2 - 4a[\theta_f - \theta_0]}}{2a}$$

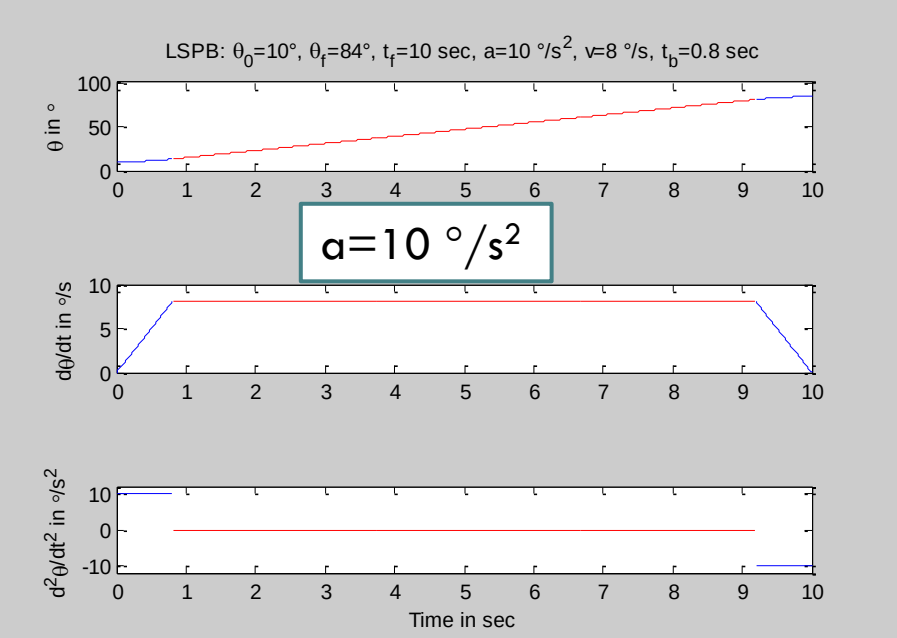
$$= \frac{t_f}{2} - \frac{\sqrt{a^2 t_f^2 - 4a[\theta_f - \theta_0]}}{2a}$$

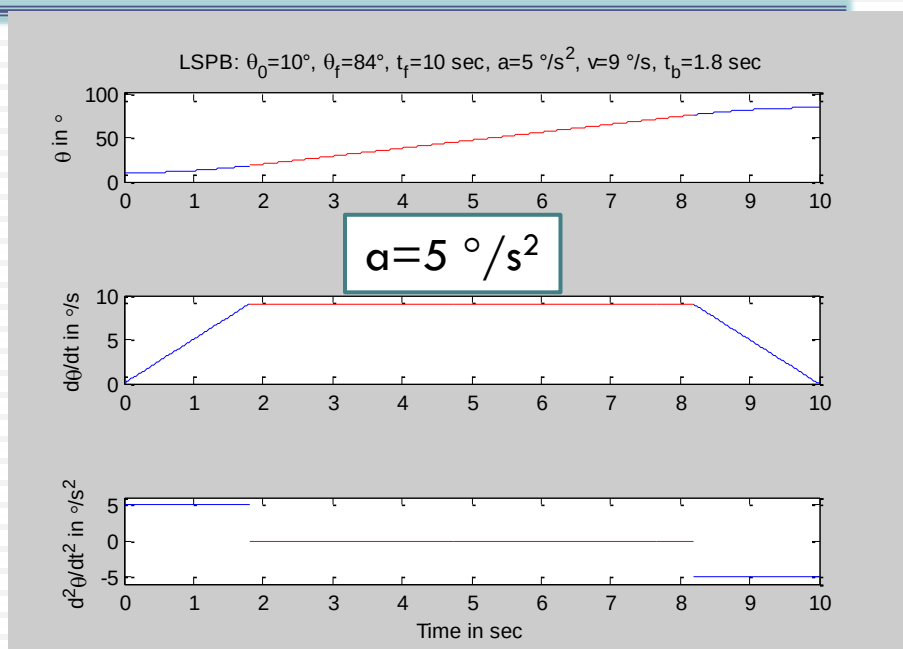
  - Intuitively, from the diagram if $t_b \geq \frac{t_f}{2}$, then a viable solution ceases to exist!! or $a^2 t_f^2 - 4a[\theta_f - \theta_0] > 0$

$$\therefore \qquad a \geq \frac{4[\theta_f - \theta_0]}{t_f^2} \qquad (6.9)$$

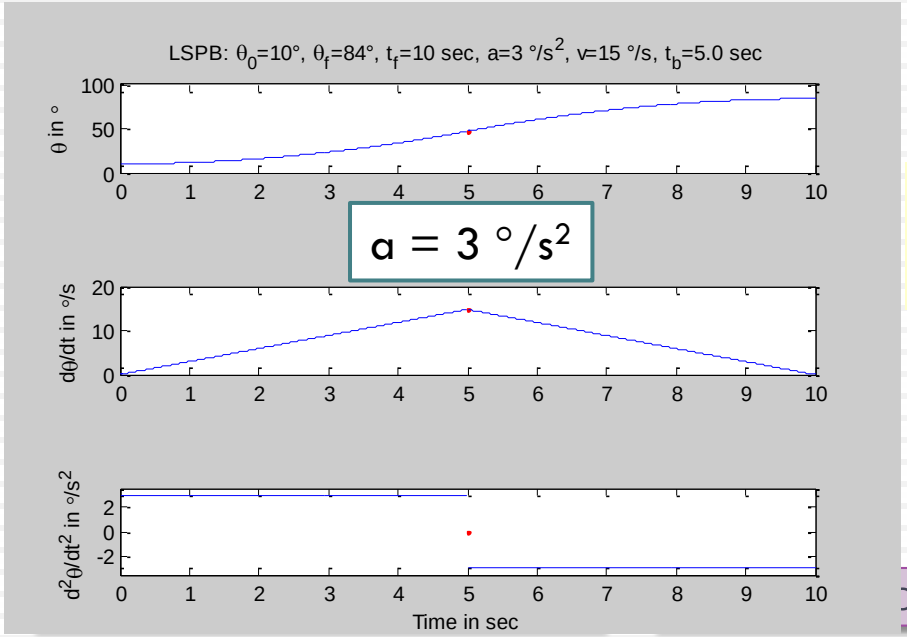
- At the point where the equality holds ($a = \frac{4[\theta_f - \theta_0]}{t_f^2}$), the linear segment is of length zero. This is known as a bang-bang parabolic blend!!

6.3.3 A Linear Segment With Parabolic Blends





A Bang-Bang Parabolic Blend



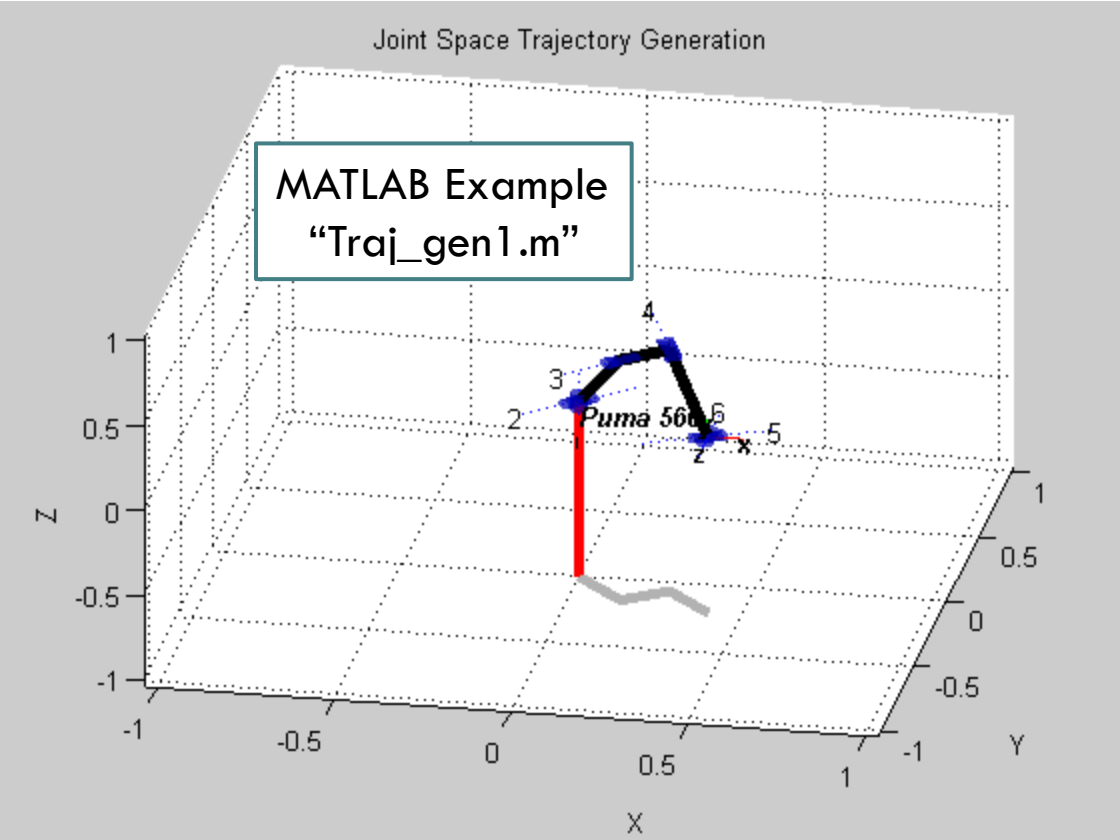
$$a \geq \frac{4[\theta_f - \theta_0]}{t_f^2} = 2.96\,°/s^2$$

6.4 Cartesian Space Schemes

- **Now specify the path in term of the tool pose**
- **Can describe straight line Cartesian trajectories**
- **Both position and orientation**
- **Requires an inverse kinematic computation at each point**
- **Describing the orientation can be challenging**
  - Euler angles?
  - RPY angles?
  - Rotation matrix?
  - Angle/axis representation?

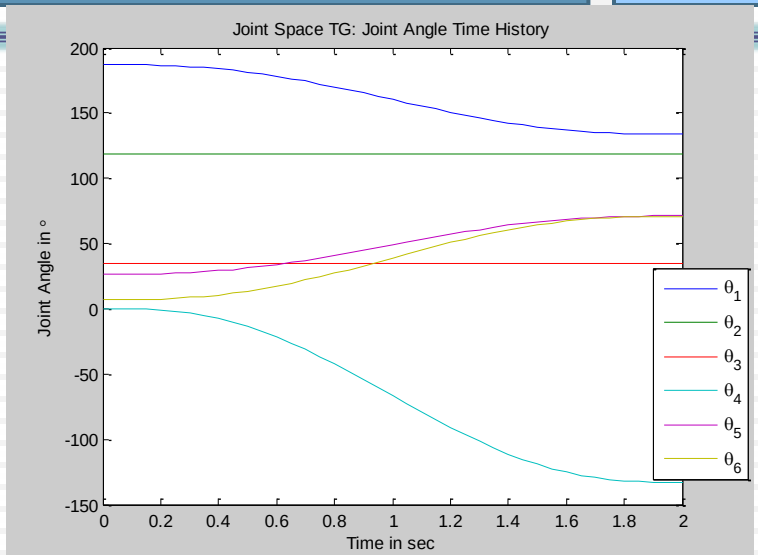
A Puma 560 example

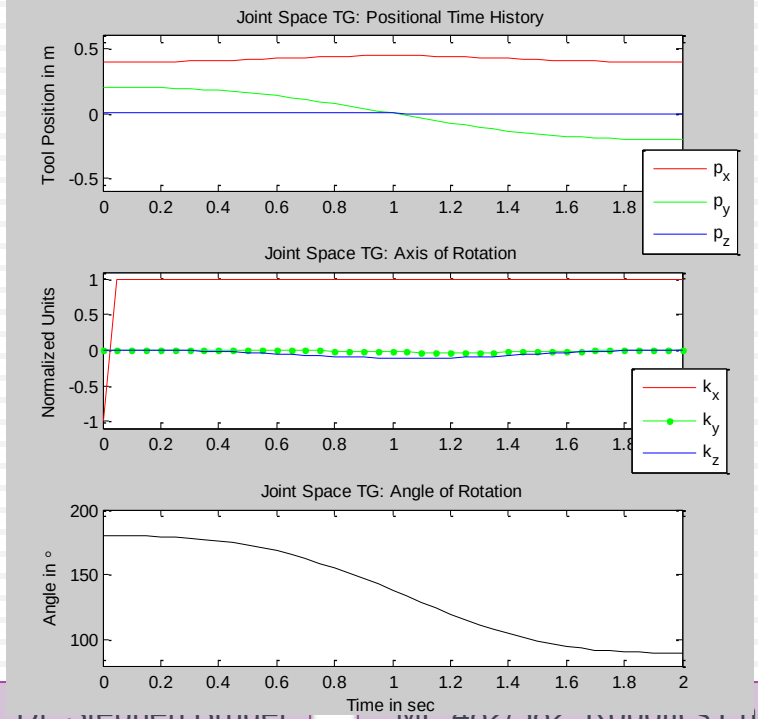
- Compare a Joint space vs Cartesian space trajectory

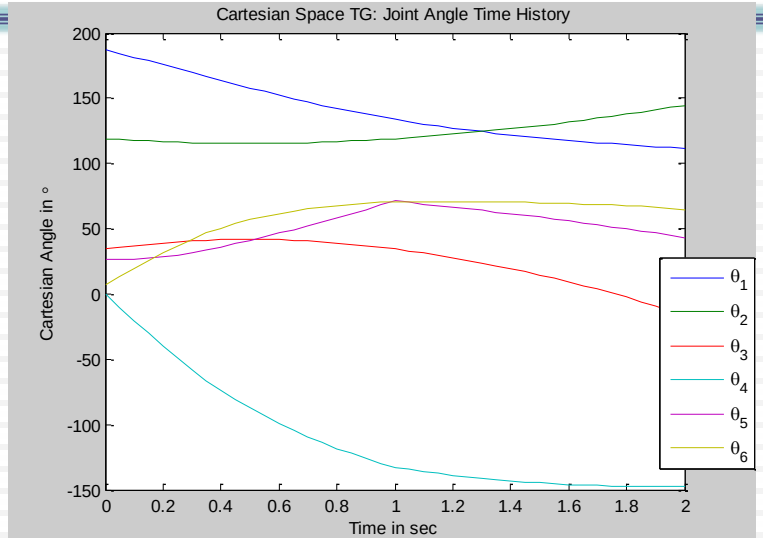


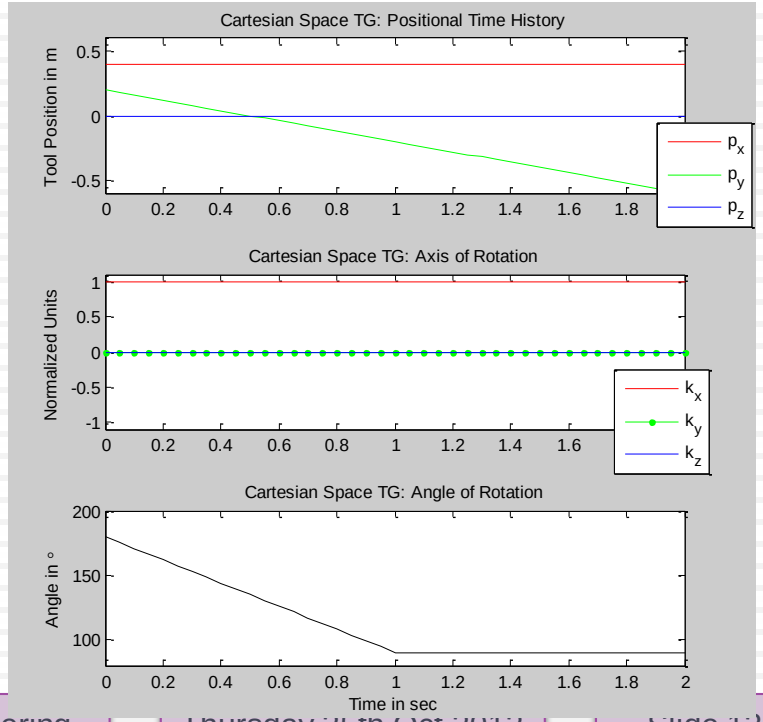
6.4 An Example: Joint vs Cartesian

Joint Space Trajectory Generation









Cartesian Space Trajectory Generation